KIT Department of Informatics
Institute for Anthropomatics and Robotics (IAR)
High Performance Humanoid Technologies (H^2T)

Robotics I, WS 2024/2025

Exercise Sheet 3

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Exercise 1 (Inverse Kinematics)

Given is a SCARA robot with one translational joint d_1 and two rotational joints θ_2 und θ_3. The forward kinematics is determined by:

$$\mathbf{f}(\mathbf{q}) = \begin{pmatrix} -500\sin(\theta_2)\cos(\theta_3) - 500\cos(\theta_2)\sin(\theta_3) - 500\sin(\theta_2) \\ 500\cos(\theta_2)\cos(\theta_3) - 500\sin(\theta_2)\sin(\theta_3) + 100 + 500\cos(\theta_2) \\ d_1 \end{pmatrix}.$$

The robot's configuration is given by $\mathbf{q} = (d_1, \theta_2, \theta_3)^\top$.

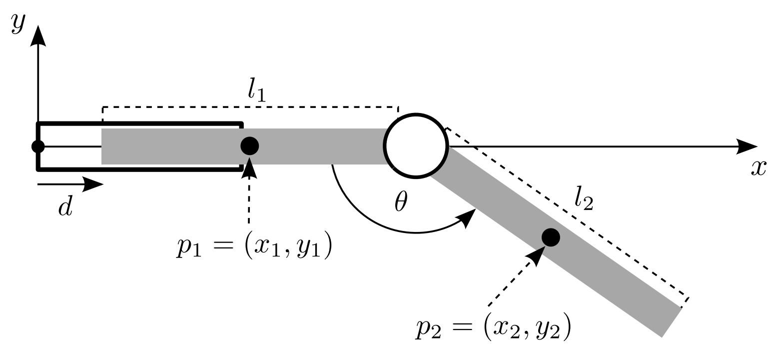
The joint angle velocity, based on the inverse kinematics is given by:

$$\dot{\mathbf{q}} = J^{-1}(\mathbf{q})\,\dot{\mathbf{x}}$$

1. Determine the inverse Jacobian $J^{-1}(\mathbf{q})$ for the given SCARA robot. The orientation of the robot can be ignored in this exercise.
2. Determine the joint angular velocity $\dot{\mathbf{q}}$, which results in an end-effector velocity $\dot{\mathbf{x}} = (1000, 0, 0)^\top$ at the state $\mathbf{q} = \left(1, 0, \frac{\pi}{2}\right)^\top$,
3. Which state results in singularities?

Exercise 2 (Lagrangian dynamic modeling)

Given is the following robotic system with two segements. The first segment s_1 has a length l_1 and mass m_1 and is connected to the base by a translational joint. The second segment s_2 has a length l_2 and a mass m_2, and is connected to s_1 by a rotational joint. For simlicity it is assumed that the center of mass $\mathbf{p_1} = (x_1, y_1)^T$ of s_1 is in the middle of the segment. Analogously this assuption is made for s_2 with the center of mass at $\mathbf{p_2} = (x_2, y_2)^T$. Gravity acts in the negative y-direction. The robot's configuration can be described by $\mathbf{q} = (d, \theta)^\top$.



With l_1, l_2 and $\mathbf{q}$ the positions of $\mathbf{p}_1$ and $\mathbf{p}_2$ can be described as follows:

$$\begin{aligned} x_1 &= \frac{1}{2} l_1 + d \\ y_1 &= 0 \\ x_2 &= l_1 + d - \frac{1}{2} l_2 \cos(\theta) \\ y_2 &= -\frac{1}{2} l_2 \sin(\theta) \end{aligned}$$

Model the dynamics of the given robot system with the method of Lagrange.

Proceed as follows and determine the following paramters:

1. The kinetic energy for both joints,
2. the potential energy for both joints,
3. the Lagrange-function.

Combine the results of the individual calculation steps into the equation of motion.

Exercise 3 (Python Introduction)

Solve the recent tasks using the Robotics Toolbox `https://petercorke.com/toolboxes/robotics-toolbox`. Therefore we created example repositories available at `https://git.h2t.iar.kit.edu/teaching/code/robotics-i/`. It contains a jupyter notebook which allows you to interactively solve the tasks in python. For installation follow the install instrctions provided in the README.

Start the notebook, reproduce the results for tasks 1-5 of exercise 1 and play around with the parameters.

Alternatives (will not be discussed in the exercise):

1. If you are already more familiar with python, install the toolboxes python package following the instruction on `https://github.com/petercorke/robotics-toolbox-python` and use your favorite python IDE.
2. If you are more familiar with matlab you can also use the matlab package provided here: `https://petercorke.com/toolboxes/robotics-toolbox/`